Structural Design of Reinforced Concrete Staircase According to Eurocode 2

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1.0 Introduction

In multi-storey buildings, ramps, elevators, escalators, and stairs are often employed to facilitate vertical circulation. Circulation refers to the movement of people and goods between interior spaces in buildings and to entrances and exits. Stairs are important building elements that are used to provide vertical circulation and access across different floor levels in a building. It is also recommended that when an access height exceeds 600mm, a staircase should be provided. In modern architecture, stairs are designed to be aesthetically pleasing, and this contributes immensely to the interior beauty of a house. There are different types of stairs with different configurations. For stairs, the recommended slope for comfort is 27°, but for practical purposes, this can sometimes be extended to 35°.

Types of stairs

Generally, stairs are usually of the following types:

- straight
- circular
- curved or spiral or
- a combination of the above mentioned types.

**Straight stairs** are stairs along which there is no curvature or change in direction on any flight between two successive floors or levels. There are several possible arrangements of straight stairs. For example, they may be arranged in a **straight run** with a single flight between floors, or a series of flights without change in direction.

Also, straight stairs may permit a change in direction at an immediate landing. When the stairs require a complete reversal of direction, they are called **parallel stairs or half landing stairs** (turning through 180°). When successive flights are at an angle to each other, (usually 90°), they are called **angle stairs or quarter turn stairs**. In addition, straight stairs may be classified as **scissors stairs** when they comprise a pair of straight runs in opposite directions and are placed on opposite sides of a wall.

**Circular stairs** when viewed from above appear to follow a circle with a single centre of curvature and large radius.

**Curved stairs** when viewed from above appear to follow a curve with two or more centres of curvature, such as an ellipse.

**Spiral stairs** are similar to circular stairs except that the radius of curvature is small and the stairs may be supported by a column.
1.1 Components of a staircase

**Flight**: A series of steps extending from floor to floor, or from a floor to an intermediate landing or platform.
Guard: Protective vertical barrier along edges of stairways, balconies, and floor openings.

Landings (platforms): Used where turns are necessary or to break up long climbs. Landings should be level, as wide as the stairs, and at least 1000mm long in the direction of travel.

Step: Combination of a riser and the tread immediately above.

Rise: Distance from floor to floor.

Run: Total length of stairs in a horizontal plane, including landings.

Riser: Vertical face of a step. Its height is generally taken as the vertical distance between treads.

Tread: Horizontal face of a step. Its width is usually taken as the horizontal distance between risers.

Nosing: Projection of a tread beyond the riser below.

Soffit: Underside of a stair.

Railing: Framework or enclosure supporting a handrail and serving as a safety barrier.

Baluster: Vertical member supporting the handrail in a railing.

Balustrade: A railing composed of balusters capped by a handrail.

Handrail: Protective bar placed at a convenient distance above the stairs for a handhold.

1.2 Brief ideas on selection of staircase dimensions

Headroom
Ample headroom should be provided not only to prevent tall people from injuring their heads, but to give a feeling of spaciousness. A person of average height should be able to extend his hand forward and upward without touching the ceiling above the stairs. Minimum vertical distance from the nosing of a tread to overhead construction should preferably never be less than 2100mm.

Stairway Width
Width of a stairway depends on its purpose and the number of persons to be accommodated in peak hours or emergencies. Also there are building codes that regulate the geometric design of stairways. The following can be used as guidelines;

For residential flats between two stories to four storeys, use a minimum width of 900mm, for flats more than 4 storeys, use width of 1000mm.

For public buildings of under 200 persons per floor, use width of 1000mm, for buildings between 200 – 400 persons per floor, use a width of 1500mm. For over 400 persons, use width between 1500 – 3000mm. However, when the width of a stair way exceeds 1800mm, it is necessary to divide it using handrails.

Step Sizes
Risers and treads generally are proportioned for comfort and to meet accessibility standards for the handicapped, although sometimes space considerations control or the desire to achieve a monumental effect, particularly for outside stairs of public buildings. Treads should be at least 250mm, exclusive of nosing. The most comfortable height of riser is 175mm. Risers less than 100mm and more than
200mm high should not be used. The steeper the slope of the stairs, the greater the ratio of riser to tread. In design of stairs, account should be taken of the fact that there is always one less tread than riser per flight of stairs. No flight of stairs should contain less than three risers.

1.3 Structural Design of stairs

These theoretical procedures employed in the structural analysis of stairs is the concept of an idealised line structure and, when detailing the reinforcement for the resulting stairs, additional bars should be included to limit the formation of cracks at the points of high stress concentration that inevitably occur. The ‘three dimensional’ nature of the actual structure and the stiffening effect of the triangular tread areas, both of which are usually ignored when analysing the structure, will result in actual stress distributions that differ from those calculated, and this must be remembered when detailing (Reynolds et al, 2008). The typical nature of internal stresses induced in a simply supported straight flight stair and reinforcement pattern is as shown in Figure 1.4.

![Figure 1.4: Structural behaviour of a reinforced concrete straight flight staircase](image)

Simple straight flights of stairs can span either transversely (i.e. across the flight) or longitudinally (i.e. along the flight). When spanning transversely, supports must be provided on both sides of the flight by either walls or stringer beams. In this case, the waist or thinnest part of the stair construction need be no more than 60 mm thick say, the effective lever arm for resisting the bending moment being about half of the maximum thickness from the nose to the soffit, measured at right angles to the soffit. When the stair spans longitudinally, deflection considerations can determine the waist thickness. In principle, the design requirements for beams and slabs apply also to staircases, but designers cannot be expected to determine the deflections likely to occur in the more complex stair types. BS 8110 deals only with simple types, and allows a modified span/effective depth ratio to be used. The bending moments should be calculated from the ultimate load due to the total weight of the
stairs and imposed load, measured on plan, combined with the horizontal span. Stresses produced by the longitudinal thrust are small and generally neglected in the design of simple systems.

2.0 Sample Design

A section of a staircase is as shown below. The width of the staircase is 1160mm. We are expected to carry out a full structural analysis and design of the staircase according to EC2 using the following data; Density of concrete = 25 KN/m$^3$; Fck = 30 N/mm$^2$; Fyk = 460 N/mm$^2$; Concrete cover = 25mm; Imposed load on staircase (q$_{ik}$) = 4 KN/m$^2$ (category C3)

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Figure 1.5: Elaborate reinforcement pattern in stair case

Figure 1.6: Sectional view of staircase to be analysed
2.1 Structural idealisation

We are going to analyse the staircase as a simple statically indeterminate frame pinned at both supports. This is shown in the Figure 1.7 below;

![Figure 1.7: Structural idealisation of the staircase in Figure 1.6](image)

2.2 Loading

Thickness of waist and landing = 200 mm

Depth of riser = 150mm

**Load actions on the stairs**

Concrete self weight (waist area) = 0.2 \times 25 = 5 \text{ KN/m}^2 \text{ (normal to the inclination)}

Stepped area = \frac{1}{2} \times 0.15 \times 25 = 1.875 \text{ KN/m}^2 \text{ (global vertical direction)}

Finishes (say) = 1.2 \text{ KN/m}^2

We intend to apply all gravity loads purely in the global y-direction, therefore we convert the load at the waist of the stair from local to global direction by considering the angle of inclination of the flight area to the horizontal;

$$\alpha = \tan^{-1} \left( \frac{1.2}{1.75} \right) = 34.438989^\circ$$

Therefore UDL from waist of the stair in the global direction is given by = (5 \times \cos 34.438989) = 4.124 \text{ KN/m}^2

\therefore \text{ Total dead load on flight area (gk) = 4.124 + 1.875 + 1.2 = 7.199 KN/m}

Variable load on staircase (qk) = 4 \text{ KN/m}^2

The load on the flight area at ultimate limit state = 1.35gk + 1.5qk

n = 1.35(7.199) + 1.5(4) = 15.719 \text{ KN/m}^2

On the landing;

\begin{align*}
gk &= 5 + 1.2 = 6.2 \text{ KN/m}^2; \\
qk &= 4 \text{ KN/m}^2
\end{align*}
The load on the landing at ultimate limit state = 1.35gk + 1.5qk

\[ n = 1.35(6.2) + 1.5(4) = 14.370\text{KN/m}^2 \]

![Diagram of staircase](image1)

Figure 1.8: The staircase with applied ultimate limit state load

### 2.3 Internal Stresses Diagram

![Diagram of bending moment](image2)

Figure 1.9: Bending moment diagram due to externally applied load

![Diagram of shear force](image3)

Figure 2.0: Shear force diagram due to externally applied load
Axial forces of 9.431 KN induced in the flight section have been neglected in this design.

2.4 Structural Design Equations
From EC2 singly reinforced concrete stress block;
\[ M_{Rd} = Fc z \]  \hspace{1cm} (1)
\[ Fc = \frac{0.85 f_{ck}}{1.5} \times 0.8 xb \ ; \ z = d - 0.4x \]  \hspace{1cm} (2)

Clause 5.6.3 of EC2 limits the depth of the neutral axis to 0.45d for concrete class less than or equal to C50/60. Therefore for an under reinforced section (ductile);
\[ x = 0.45d \]  \hspace{1cm} (3)

Combining equation (1), (2) and (3), we obtain the ultimate moment of resistance \( M_{Rd} \)
\[ M_{Rd} = 0.167 f_{ck} b d^2 \]  \hspace{1cm} (4)

Also from the reinforced concrete stress block;
\[ M_{Ed} = Fsz \]  \hspace{1cm} (5)
\[ F_s = \frac{f_{yk}}{1.15} A_{s1} \]  \hspace{1cm} (6)

Substituting equ (6) into (5) and making \( A_{s1} \) the subject of the formular;
\[ A_{s1} = \frac{M_{Ed}}{0.87 f_{yk} z} \]  \hspace{1cm} (7)

The lever arm \( z \) in EC2 is given;
\[ z = d \left[ 0.5 + \sqrt{(0.25 - 0.882K)} \right] \]  \hspace{1cm} (8)
where \( K = \frac{M_{Ed}}{f_{ck} b d^2} \)  \hspace{1cm} (9)

For doubly reinforced sections;

Area of compression reinforcement \( A_{S2} = \frac{M_{Ed} - M_{Rd}}{0.87 f_{yk}(d - d_z)} \)  \hspace{1cm} (10)

Area of tension reinforcement \( A_{s1} = \frac{M_{Rd}}{0.87 f_{yk} z} + A_{S2} \)  \hspace{1cm} (11)

Where \( z = d \left[ 0.5 + \sqrt{(0.25 - 0.882K')} \right] \) where \( K' = 0.167 \)

Check for deflection (Clause 7.4.2)

The limiting basic span/ effective depth ratio is given by;
\[ \frac{L}{d} = K \left[ 11 + 1.5 \sqrt{f_{ck} \frac{\rho_0}{\rho}} + 3.2 \sqrt{f_{ck} \left( \frac{\rho_0}{\rho} - 1 \right)^{3/2}} \right] \text{ if } \rho \leq \rho_0 \]  \hspace{1cm} (12)
\[ \frac{L}{d} = K \left[ 11 + 1.5 \sqrt{f_{ck} \frac{\rho_0}{\rho}} + \frac{1}{12} \sqrt{f_{ck} \left( \frac{\rho_0}{\rho} \right)^{1/2}} \right] \text{ if } \rho > \rho_0 \]  \hspace{1cm} (13)

Where;
L/d is the limiting span/depth ratio
K = Factor to take into account different structural systems
ρ₀ = reference reinforcement ratio = 10⁻³\sqrt{f_{ck}}
ρ = Tension reinforcement ratio to resist moment due to design load
ρ' = Compression reinforcement ratio

Shear design

In EC2, the concrete resistance shear stress without shear reinforcement is given by;
\[ V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100\rho_1 f_{ck}^{1/3} + k_1 \sigma_{cp})bw.d \geq (V_{min} + k_1 \sigma_{cp})] \]  
\[ C_{Rd,c} = 0.18/\gamma_c ; k = 1+\sqrt{\frac{200}{d}} < 0.02 \text{ (d in mm)} ; \rho_1 = \frac{A_{S1}}{bd} < 0.02 \text{ (In which } A_{S1} \text{ is the area of tensile reinforcement which extends } \geq (l_{bd} + d) \text{ beyond the section considered; } V_{min} = 0.035k^2f_{ck}^{1/2} \]
K₁ = 0.15; \sigma_{cp} = N_{Ed}/Ac < 0.2f_{cd} \text{ (Where } N_{Ed} \text{ is the axial force at the section, } Ac = \text{ cross sectional area of the concrete), } f_{cd} = \text{ design compressive strength of the concrete.}

Design of the span

A little consideration will show that it is best to use the design moment \( M_{Ed} = 41.119 \text{ KNm} \) to design the entire stairs.
\( M_{Ed} = 41.119 \text{ KNm} \)
d = h – Cc – ϕ/2
Assuming ϕ12mm bars will be employed for the construction
d = 200 – 25 – 6 = 169mm; b = 1000mm \text{ (designing per unit width)}

\[ k = \frac{M_{Ed}}{f_{ck} \cdot bd^2} = \frac{41.119 \times 10^6}{30 \times 1000 \times 169^2} = 0.0479 \]

Since k < 0.167 No compression reinforcement required
\[ z = d\left[0.5 + \sqrt{(0.25 - 0.882k)}\right] = z = d\left[0.5 + \sqrt{(0.25 - 0.882(0.0479)}\right] = 0.95d \]
\[ A_{S1} = \frac{M_{Ed}}{0.87f_{yk}z} = \frac{41.119 \times 10^6}{0.87 \times 460 \times 0.95 \times 169} = 639.96 \text{ mm}^2/m \]

Provide Y12mm @ 150mm c/c BOT (\( AS_{prov} = 753 \text{ mm}^2/m \))

To calculate the minimum area of steel required;
\( f_{ctm} = 0.3 \times f_{ck}^{2/3} = 0.3 \times 30^{2/3} = 2.896 \text{ N/mm}^2 \) \text{ (Table 3.1 EC2)}

\[ A_{Smin} = 0.26 \times \frac{f_{ctm}}{f_{yk}} \times b \times d = 0.26 \times \frac{2.896}{460} \times 1000 \times 169 = 276.631 \text{ mm}^2/m \]
Check if \( A_{Smin} < 0.0013 \times b \times d \text{ (219.7 mm}^2/m)\)
Since, \( A_{Smin} = 276.631 \text{ mm}^2 \), the provided reinforcement is adequate.
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Provide Y12 @ 250mm c/c as distribution bars (AS\text{prov} = 452 mm²/m)

**Check for deflection;**

K = 1.0 for simply supported beams and slab

\[ \rho = \frac{A_s}{bd} = \frac{753}{1000 \times 169} = 0.004455 < 10^{-3} \sqrt{30} \]

Since \( \rho < \rho_0 \)

\[ \frac{L}{d} = K \left[ 11 + 1.5 \sqrt{f_{ck}} \frac{\rho_a}{\rho} + 3.2 \sqrt{f_{ck}} \left( \frac{\rho_a}{\rho} - 1 \right)^{3/2} \right] \]

\[ \frac{L}{d} = 1.0 \left[ 11 + 1.5 \sqrt{30} \times \frac{0.005477}{0.004455} + 3.2 \sqrt{30} \left( \frac{0.005477}{0.004455} - 1 \right)^{3/2} \right] = 1.0 (21.1 + 1.9258) = 23.0258 \]

Modification factor \( \beta_s = \frac{310}{\sigma_s} \)

\[ \sigma_s = \frac{310 f_{yk} A_{req}}{500 A_{prov}} = \frac{310 \times 460 \times 639.96}{500 \times 753} = 242.358 \text{ N/mm}^2 \]

\[ \beta_s = \frac{310}{242.358} = 1.2789 \]

Taking the distance between supports as the effective span, \( L = 4.35 \)m

The allowable span/depth ratio = \( \beta_s \times 23.0258 = 1.2789 \times 23.0358 = 29.460 \)

Actual deflection \( \frac{L}{d} = \frac{4350}{169} = 25.739 \)

Since 25.739 < 29.460, deflection is ok.

**Shear Design**

Ultimate shear force \( V_{Ed} = 35.358 \) KN

\[ V_{Rd,c} = [C_{Rd,c} \times k \times (100 \rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp} b w . d \geq (V_{min} + k_1 \sigma_{cp}) b w . d \]

\[ C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12 \]

\[ k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{169}} = 2.087 > 2.0, \text{ therefore, } k = 2 \]

\[ V_{min} = 0.035 k^2 f_{ck} \frac{1}{\gamma_c} = V_{min} = 0.035 \times (2)^{\frac{3}{2}} \times (30)^{\frac{1}{2}} = 0.5422 \text{ N/mm}^2 \]

\[ \rho_1 = \frac{A_s}{bd} = \frac{753}{1000 \times 169} = 0.004456 < 0.02; \text{ K_1 = 0.15} \]

\[ \sigma_{cp} = N_{Ed}/A_c < 0.2fcd \text{(Where } N_{Ed} \text{ is the axial force at the section, } A_c = \text{ cross sectional area of the concrete), fcd = design compressive strength of the concrete.) Take } N_{Ed} = 0 \]

\[ V_{Rd,c} = [0.12 \times 2(100 \times 0.004456 \times 30 \times 753)] 1000 \times 169 = 96258.7969 \text{ N = 96.258 KN} \]

Since \( V_{Rd,c} \) (96.258 KN) > \( V_{Ed} \) (35.358 KN), no shear reinforcement is required. Shear is ok.
Detailing sketches

Figure 2.1: Plan view of the reinforcement of the staircase

Figure 2.2: Section view (section 2-2) of the reinforcement of the staircase